THE COMPRESSION WAVE DURING IMPACT ON A FREE SUSPENSION SURFACE

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The initial chracteristics of a plane wave being generated during massive body entrance into a suspension are determined. The pressures occurring in the wave are sufficient for formation of hydration products imparting binding properties to a water-ceramic suspension.

A compression wave that is propagated deep into a fluid from its free surface and later results in the development of a nonlinear acoustic wave occurs during shock entrance of a body in a fluid at velocities considerably smaller than the speed of sound. However, even before the body succeeds in being retarded substantially, pressures occur in the wave that are significantly higher than in ordinary sonic waves. At this stage it is perfectly natural to neglect compressibility of the fluid. But known shock problems of the hydrodynamics of an incompressible fluid permit describing just the total pressure pulse received by the fluid and this is not the pressure itself [1]. Consequently, the compressibility at the earliest stage of wave generation must already be taken into account to obtain more detailed information about the dynamics of the pressure change and about the maximal values it attained.

Besides the possible applications to a number of nonlinear acoustics problems, the problem of creating high pressures on the boundaries of interphasal interface is of interest in connection with working out and optimizing the technology of preparing water-ceramic suspensions with binder properties used in annealing-free production processes for ceramic articles [2]. It is established empirically that after mechanical treatment of such suspensions in even relatively weakly energy-stressed tumbling barrels, they acquire the capacity to be shaped without insertion of special binding admixtures, which is of exceptional importance for modern technologies, that are quite responsive to purity of materials. The physicochemical mechanisms for the appearance of intrinsic binders in a suspension during its mechanical chemical activation were studied and consist in the accumulation therein of water dissociation products and in the origination of appropriate hydrate coatings on particles stimulating the polycondensation process that indeed results in the formation of strong structural bonds during dehydration. However, high stresses at the particle surfaces close to the breaking point (10^6-10^7 Pa) present at time intervals greatly exceeding the period of molecular vibrations ($\sim 10^{-12}$ sec) are necessary for the realization of such a process. The loadings occurring during the motion of grinding bodies at a velocity of around 1 m/sec in the bulk of a suspension, are approximately 10^3 Pa, i.e., several tens of orders smaller than that needed. Consequently, they cannot be considered as a factor explaining the suspensions acquiring binding properties.

Higher stresses can be produced under the impacts of grinding surfaces on the free surface of a suspension. A strongly reduced estimate of their magnitude can be obtained by referring the pressure pulse following from the solution of the appropriate shock problem for an incompressible fluid to the characteristic time interval. However, as is shown in [3], such a rough estimate is already sufficient for the deduction that the necessary pressures can be produced in quite thin films on surfaces of individual particles under the impact of a massive body. In this case the formation of hydration products is possible only in a very limited number of particles that are directly on the free surface of the suspension at the time of impact. Consequently, refinement of the results [3] and determination of the maximal pressure values over the whole near-surface layer of the suspension is of interest.

Let us consider the motion stimulated during the impact of a flat body on the free surface of a finely-dispersed suspension of hard particles in a weakly-compressible fluid.

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 0022-0841/91/6002-0160$12.50

For simplicity, we limit ourselves to analyzing the one-dimensional problem at the time directly after impact, when the suspended particles have still not succeeded in being accelerated, when the suspended particles still do not succeed in being accelerated because of interphasal interaction while the inertial forces significantly exceed all the rest. Then the velocity of the dispersed phase can be considered equal to zero while its bulk content is unchanged. Moreover, it is allowable to neglect the Stokes viscous interaction force as compared with the forces related to deceleration of the apparent fluid masses by the particles and to the Bass effect and also by expecting high maximal values of the pressure, not to take account of the ordinary dynamic pressure of the order of $\rho V_0{}^2$.

The mass and momentum conservation equations of the continuous phase of the suspension are written under the mentioned assumptions in the form

$$\frac{\partial\rho}{\partial t}+\rho\frac{\partial v}{\partial x}=0,$$

$$\varepsilon\rho\frac{\partial v}{\partial t}=-\frac{\partial p}{\partial x}-(1-\varepsilon)\rho\left[A\frac{\partial v}{\partial t}+B\int_0^t\frac{\partial v}{\partial\tau}\frac{d\tau}{\sqrt{t-\tau}}\right], \quad (1)$$

where for moderately concentrated suspensions [4]

$$A=\frac{3}{2}\varepsilon\left[1+(1-\varepsilon)\left(\frac{1}{2}+\frac{15}{4}M(\varepsilon)\right)\right],$$

$$B=\frac{9}{2}[\varepsilon M(\varepsilon)]^{1/2}\left(\frac{\mu}{\pi\rho a^2}\right)^{1/2},\quad M(\varepsilon)=[1-(5/2)(1-\varepsilon)]^{-1}. \quad (2)$$

The extension of higher concentrations to the suspension is possible if an experimentally found dependence of the relative viscosity of the suspension on its bulk concentration $1-\varepsilon$ is used as $M(\varepsilon)$ in the first two formulas in (2).

Keeping in mind the model purposes, we approximate the dependence $\rho(p)$ by a linear function such that

$$\frac{\partial\rho}{\partial t}\approx K\frac{\partial p}{\partial t},\quad K=\frac{1}{c^2}, \quad (3)$$

where c is some effective velocity that agrees with the speed of sound in a pure fluid for not too high p and because of the smallness of the adiabatic compressibility factor K we set ρ equal to a constant in (1) and (2). This quantity certainly does not have a relation to the speed of sound in the suspension.

The equation governing the change in velocity of the impacting body follows from Newton's second law

$$m\,dV/dt=-(p|_{x=0}-p_0). \quad (4)$$

The boundary and initial conditions imposed on the solution of the written equations have the form

$$v=0,\ p=p_0\ \text{ for }\ t=0\ \text{and}\ x\to\infty,$$

$$v=V\ \text{ for }\ x=0,\quad V=V_0\ \text{ for }\ t=0. \quad (5)$$

Since the problem in the approximation under consideration is linearized successfully, it is natural to use the Laplace transform (with transformation parameter s). Taking (3) into account there follows from (1) and (4)

$$c^{-2}(sp-p_0)=-\rho\,dv/dx,\quad [\varepsilon+(1-\varepsilon)A]\rho sv=-dp/dx-B\rho\sqrt{\pi s}v,$$

$$m(sV-V_0)=-p|_{x=0}+p_0/s$$

(we retain the same symbols for the transforms of the desired functions as for the originals). Eliminating p here we obtain

$$\frac{d^2v}{dx^2}=-\lambda^2 v,\quad \lambda^2=\left[\alpha+\beta\left(\frac{\mu}{\rho a^2 s}\right)^{1/2}\right]\frac{s^2}{c^2}, \quad (6)$$

$$p = \frac{p_0}{s} - \frac{\rho c^2}{s}\frac{dv}{dx}, \quad V = \frac{V_0}{s} + \frac{\rho c^2}{ms^2}\frac{dv}{dx}\bigg|_{x=0},$$

where the concentration functions have been introduced

$$\alpha = \varepsilon + (1-\varepsilon) A, \quad \beta = (9/2)(1-\varepsilon)[\varepsilon M(\varepsilon)]^{1/2}. \tag{7}$$

The solution of the equations for v under the conditions v = V for x = 0, v → 0 as x → ∞, that follow from (5) result in the formulas

$$v = \frac{V_0}{s(1+\rho c^2\lambda/ms^2)} e^{-\lambda x}, \quad p = \frac{p_0}{s} + \frac{\rho c^2 V_0 \lambda}{s^2(1+\rho c^2\lambda/ms^2)} e^{-\lambda x}, \tag{8}$$

when the expression for V in (6) is taken into account.

The asymptotics of the expressions for v and p that correspond to small times after impact are obtained from (8) for large s. Taking account of the definition of λ in (6), we obtain for $t^{-1} \sim s \gg (\beta/\alpha)^2\mu/\rho a^2$

$$p = \frac{p_0}{s} + \frac{p_m\gamma}{s}\exp\left(-\frac{x\sqrt{\alpha}\gamma s}{c}\right), \quad \gamma = 1 + \frac{\beta}{2\alpha}\left(\frac{\mu}{\rho a^2 s}\right)^{1/2}, \tag{9}$$

where the amplitude value of the pressure has been introduced

$$p_m = \sqrt{\alpha}\rho c V_0 = \alpha\rho u V_0, \quad u = c/\sqrt{\alpha}. \tag{10}$$

If components proportional to β are generally neglected, then the original of the transform (9) is an undamped wave

$$p - p_0 \approx p_m \eta(t - x/u), \tag{11}$$

propagated deep into the fluid at a velocity u somewhat less than the speed of sound in the homogeneous liquid phase.

To find the correction taking account of the damping of this pressure wave we use the theorem of lag of operational calculus

$$f(t-\tau)\eta(t-\tau) \div e^{-s\tau} f(s), \quad f(t) \div f(s)$$

and the tabulated correspondences between the transforms and the originals [5]. In the case under consideration

$$f(s) = \frac{p_m}{s}\left(1 + \frac{\beta\varkappa c}{2\alpha\sqrt{s}}\right)\exp\left(-\frac{\beta\varkappa x}{2\sqrt{\alpha}}\sqrt{s}\right),$$

$$\tau = \frac{x\sqrt{\alpha}}{c} = \frac{x}{u}, \quad \varkappa = \left(\frac{\mu}{\rho a^2 c^2}\right)^{1/2},$$

where

$$f(t) = p_m \operatorname{erfc}\left(\frac{\beta\varkappa x}{4\sqrt{\alpha t}}\right) +$$

$$+ \frac{\beta\varkappa c}{2\alpha} p_m \left\{2\sqrt{\frac{t}{\pi}}\exp\left(-\frac{\beta^2\varkappa^2 x^2}{16\alpha t}\right) - \frac{\beta\varkappa x}{2\sqrt{\alpha}}\operatorname{erfc}\left(\frac{\beta\varkappa x}{4\sqrt{\alpha t}}\right)\right\}.$$

Consequently, keeping only the principal term in (9) for the pressure transform, we obtain

$$p - p_0 \approx p_m \operatorname{erfc}\left[\frac{\beta\varkappa x}{4\sqrt{\alpha}}\left(t - \frac{x}{u}\right)^{-1/2}\right]\eta\left(t - \frac{x}{u}\right), \tag{12}$$

which is approximately valid upon satisfaction of the inequalities

$$t \ll \frac{\pi\alpha}{\beta^2\varkappa^2 c^2} \sim \frac{\rho a^2}{\mu}, \quad x \ll \frac{4\alpha^{3/2}}{\beta^2\varkappa^2 c} \sim \frac{\rho a^2}{\mu} c. \tag{13}$$

It is easy to write down analogous approximate representations for the velocities v and V also. Thus

$$v \approx V_0 \operatorname{erfc}\left[\frac{\beta\varkappa x}{4\sqrt{\alpha}}\left(t-\frac{x}{u}\right)^{-1/2}\right]\eta\left(t-\frac{x}{u}\right). \tag{14}$$

Let us confirm the validity of the approximations utilized in the analysis. Neglecting the operator $v\partial/\partial x$ as compared with $\partial/\partial t$ is adequate for $\lambda V_0 \ll s$ with λ from (6), which is equivalent to the requirement of smallness of the body velocity as compared with the speed of sound when (13) is satisfied. Neglecting the Stokes viscous force as compared with the remaining components of the interphasal interaction is perfectly justified upon satisfaction of the first inequality in (13) that actually asserts the smallness of the Bass force as compared with the inertial force associated with deceleration of the apparent masses. This inequality requires $t \ll 10^{-4}$ sec for aqueous suspensions of particles of radius $\sim 10^{-5}$ m. Not taking the deceleration of the impacting body into account is possible, as follows from (4) and (10), for $t \ll mV_0/p_m \sim m/\rho c$, which is a weaker constraint than that following from (13) under real conditions. Using the mechanics of the suspension to describe the process of the continual equations (1) is allowable in case the characteristic linear scale of the process is much greater than the particle radius. If a quantity 10^2 times smaller than the right side of the second inequality in (13) is even taken as such a scale, we arrive at the requirement $10^{-2}\ \rho a c/\mu \gg 1$ which is satisfied with very good accuracy for aqueous suspensions ($\rho/\mu \sim 10^6$ sec/m^2, $c \sim 10^3$ m/sec) of particles with a $\sim 10^{-6}$-10^{-5} m. Neglecting the involvement of particles in the motion of the wave being propagated, which generally requires smallness of the fluid density as compared with the density of the particle material turns out to be the most limiting. However, it is also allowable for obtaining ordinal estimates, as is the main purpose of this paper.

The pressure profiles in the wave described by (12) are shown in Fig. 1. If c is understood to be the speed of sound in water equal to approximately 1430 m/sec, then $\rho c V_0 = 1.43 \cdot 10^6$ Pa. The dependence of the ratio $p_m/\rho c V_0 = \sqrt{\alpha}$ on the bulk concentration of the suspension is shown in Fig. 2. It is seen that the presence of the interphasal interaction force that hinders free fluid flow results in a certain increase in the amplitude value of the pressure.

Let us estimate the local stresses directly at the suspended particle surfaces. The force acting on one particle is obtained upon dividing the interphasal interaction force in (1) by the numerical particle concentration $n = 3(1-\varepsilon)/4\pi a^3$. The stress σ on the particle surface equals, in order of magnitude, this force divided by the middle section area πa^2. On the basis of (1) and (8) we obtain for the transform of this quantity in the previous approximation

$$\sigma \approx \frac{4}{3}\left(A + B\sqrt{\frac{\pi}{s}}\right) a\rho V_0 \exp\left\{-\frac{x\sqrt{\alpha}s}{c}\left[1+\frac{\beta}{2\alpha}\left(\frac{\mu}{\rho a^2 s}\right)^{1/2}\right]\right\},$$

from which the expression for the original follows, $t > x/u$,

$$\sigma \approx \frac{4a\rho V_0 c\beta\varkappa}{3\sqrt{\pi(t-x/u)}}\left[1+\frac{A/\alpha}{4(t-x/u)}\frac{x}{u}\right]\exp\left[-\frac{\beta^2\varkappa^2 x^2}{16\alpha}\left(t-\frac{x}{u}\right)^{-1}\right]. \tag{15}$$

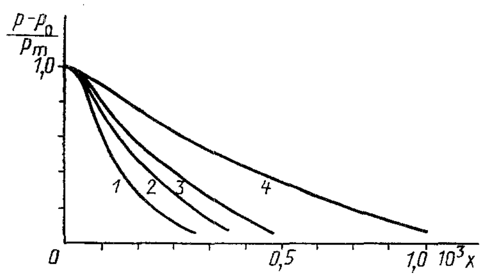

Fig. 1

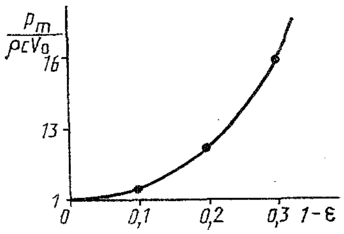

Fig. 2

Fig. 1. Relative pressure profiles in a wave at different times for an aqueous suspension with $\varepsilon = 0.8$, $a = 10^{-5}$ m for $V_0 = 1$ m/sec: 1-4) 10^6 t = 1, 2, 3, 4 sec. 10^3 x, m.

Fig. 2. Dependence of the relative maximal pressure on the suspension concentration.

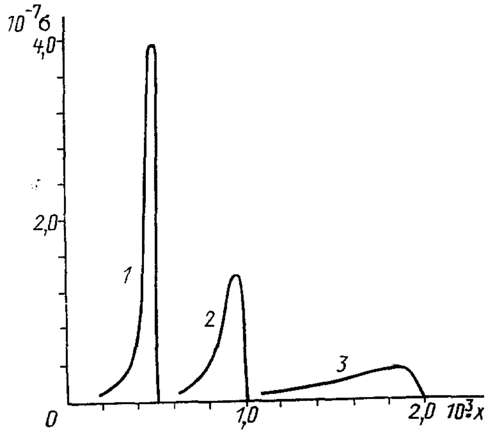

Fig. 3. Stress profiles on particle surfaces for the same parameters as for Fig. 1: 1-3) 10^6 t = 0.5, 1, 2 sec. 10^{-7} σ, Pa.

The dependences of σ on t and x are represented (see Fig. 3) for the same conditions as the curves in Fig. 1.

The stress (15) depends quite strongly on the suspension concentration which vanishes when going over to dilute suspensions. This is related to the fact that the force acting on a particle in the wave is determined by the magnitude of not the pressure itself but of its gradient. As can be seen from (12) this latter is close to zero in dilute suspensions ($\beta \approx 0$). This corresponds to replacement of the damped pressure wave (12) by the undamped wave (11). Let us note that damping within the framework of the proposed model is due entirely to the action of the Bass force but not the Stokes force and not the viscous stresses in the continuous phase of the suspension.

As follows from (12) and (15) as well as from Figs. 1 and 3, pressures and stresses are actually developed on the particle surfaces in the suspensions that achieve several MPa and act for time intervals on the order of 10^{-6} sec or more. Under practical conditions this is completely sufficient for the realization of hydration processes on particles [2, 3] in an extended near-surface layer and not only in thin films enveloping the particles on the free surface. The thickness of this layer δ is of the order of 10^{-3} m (see Fig. 3) and the quantity of particles contained therein in a computation on a free surface area S is $n\delta S$. This number is approximately $4.8 \cdot 10^6$ for the suspension displayed in Fig. 3 and for $S = 1\ cm^2 = 10^{-4}\ m^2$.

In conclusion, let us emphasize that the model developed above for involving the fluid in motion for an impact on its free surface is certainly not valid if the velocity of the impacting body is of the order of the speed of sound in the fluid (when a shock is developed and the convective parts of the total time derivatives are substantial) or exceed it significantly (when the compressibility ceases to play any role and the model proposed in [6] is apparently adequate).

Notation. a is the particle radius; A, B are coefficients defined in (2); c is the effective speed of sound; K is the adiabatic compressibility factor; m is the mass of the impacting body in a computation per unit free surface area; M(ε) is a function defined in (2); p, p_0 are the pressure and its initial value; p_m is the amplitude value of the pressure wave defined in (10); s is the Laplace transform parameter; t is the time; u is the wave velocity; v is the fluid velocity; V, V_0 are the body velocity and its initial value; x is a coordinate with origin on the free surface; α, β are functions introduced in (7); ε is the suspension porosity; η(t) is the Heaviside stepfunction; $\kappa - (\mu/\rho a^2 c^2)^{1/2}$; λ is the exponent in (6); μ, ρ are the fluid viscosity and density; σ is the characteristic stress on the particle surface.

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MASS-TRANSFER MECHANISM IN A VIBRATIONALLY FLUIDIZED CHEMICAL REACTION DISPERSION

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UDC 66.096.5

An elementary analysis is applied to the mass transport mechanism in a solid-state chemical process occurring in a vibrating finely divided mixture. The analysis is confirmed by experiment.

It is usual to conduct a solid-state process involving high-temperature chemical or physicochemical transformations in a material ground to a size level of 0.1-10 μm by means of alternating heating and grinding stages until a single-phase product is obtained. A vibrofluidized bed enables one to combine these stages [1], a key aspect being overcoming sintering, where the solution enables one to optimize the outer interactions.

1. Autohesion Particle Interaction in a High-Temperature Fluidized Bed. The charge usually belongs to group C in Geldart's classification (aspheric particles less than 20-30 μm not fluidized by gas), which coalesce into primary formations under molecular forces. In a stationary bed of chemically inert particles, one gets branched chain structures, which serve to maintain the porosity of $\varepsilon \sim 0.85$-0.95 in a stable state. When the bed is vibrated, the chains are broken and the extreme particles come together to form denser aggregates, whose size in the vibrational-force field may be described by [2]

$$D = \frac{\sigma}{\rho(1-\varepsilon)(1+K_V)g}. \quad (1)$$

That situation occurs for relative small other external forces, including hydrodynamic ones, e.g., in the vibrational treatment of powders in the mobile* state or under low vacuum ($P_0 \leq 10^4$ Pa), when the gas pressure pulsations are at a level close to $P \leq 1$ kPa [4].

Hydrodynamic forces have more effect on the size, as they occur during vibration and are external in relation to the bed. The microporous body has an unsymmetrical force characteristic, and an aggregate has a high hydrodynamic resistance and effectively damps the external gas-pressure pulsations. A positive pulsation compresses the body slightly. A negative pulsation generates tensile forces, which lead to breakup if the tensile strength is exceeded. The pressure-perturbation penetration depth is given by [5], which in our symbols is

$$H_f = \sqrt{a_0^2 \tau_v / \pi f}. \quad (2)$$

The theory shows that the gas pressure amplitude in that range is reduced by a factor e in the pores. Very high tensile forces develop along the channels, with the maximum occurring in the surfaces of the aggregates. One linearizes the pressure distribution to determine the equilibrium diameter of the particle aggregates detached from the initial body by those forces from the proportion

*Classification of vibrational fluidization mode from [3].

Kirov Urals Polytechnical Institute, Sverdlovsk. Translated from Inzhenerno-fizicheskii Zhurnal, No. 2, pp. 209-217, February, 1991. Original article submitted February 12, 1990.